GRADE 3 • MODULE 3

Multiplication and Division with Units
of 0, 1, 6–9, and Multiples of 10

Terminology:

- Even, odd (number)
- Multiple (specifically with reference to naming multiples of 9 and 10, e.g., 20, 30, 40, etc.)
- Multiplier (the factor representing the number of units)
- Product (the quantity resulting from multiplying two or more numbers together)
- Commutative Property (e.g., $2 \times 3 = 3 \times 2$)
- Distribute (with reference to the distributive property; e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$, the 3 is multiplier for each part of the decomposition)

Topic A: The Properties of Multiplication and Division

- Students study the commutativity of familiar Module 1 facts that use units of 2, 3, 4, 5, and 10. Through study they "discover" facts that they already know using units of 6, 7, 8, and 9.
    - For example, students recognize that if they know $3 \times 6 = 18$, then they also know $6 \times 3 = 18$
- They realize that they already know more than half of their facts by recognizing, for example, that if they know 2×8, they also know 8×2 through commutativity.
- Students apply the commutative property to relate 5×8 and 8×5, and then add one more group of 8 to solve 6×8 and, by extension, 8×6.

Topic B: Multiplication and Division Using Units of 6 and 7

- introduces units of 6 and 7

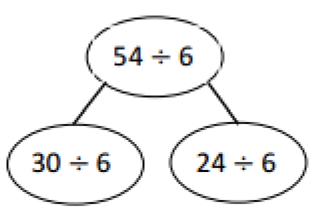
- To solve a fact using units of 7 they might count 7, 14, and then mentally add 14 + 6 + 1 to make 21. This skip-counting method utilizes make ten strategies.

- Although a formal introduction to the associative property comes in Topic C, these lessons preview the concept using addition:

  6 + 6 = 6 + 4 + 2
  18 + 6 = 18 + 2 + 4
  36 + 6 = 36 + 4 + 2
  48 + 6 = 48 + 2 + 4

- There is a formal re-introduction of the distributive property using the 5 + n pattern to multiply and divide. Students understand that multiples of 6 can be thought of as (5 + 1) × n to make 5 and 1 more groups, or 6 groups of n. Similarly, multiples of 7 can be thought of as (5 + 2) × n to make 5 and 2 more groups, or 7 groups of n.

- In division students decompose the dividend using a multiple of 5, and then add the quotients of the smaller division facts to find the quotient of the larger unknown division fact. For example:
  They decompose larger unknown facts into smaller known facts to solve. For example, 48 ÷ 6 becomes (30 ÷ 6) + (18 ÷ 6), or 5 + 3



$$\begin{aligned} 54 \div 6 &= (30 \div 6) + (24 \div 6) \\ &= 5 + 4 \\ &= 9 \end{aligned}$$

Topic C: Multiplication and Division Using Units up to 8

- Formal introduction of the associative property with a lesson on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation

- Rewriting 6 as 2 × 3 or 8 as 2 × 4 makes shifts in grouping readily apparent (see example below), and also utilizes familiar factors 2, 3, and 4 as students learn the new material. The following strategy may be used to solve a problem like 8 × 5:
  - $8 \times 5 = (4 \times 2) \times 5$
  - $8 \times 5 = 4 \times (2 \times 5)$
  - $8 \times 5 = 4 \times 10$

- They understand division as both a quantity divided into equal groups and an unknown factor problem for which—given the large size of units—skip-counting to solve can be more efficient than dividing

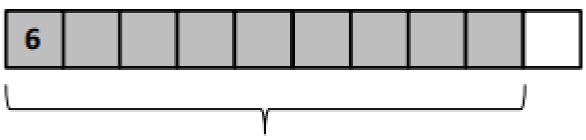
- Students use the 5 + n pattern as a strategy for solving multiplication and division problems using units of 8 with the distributive property. They learn that multiples of 8 can be thought of as $(5 + 3) \times n$. In division problems they practice decomposing the dividend using multiples of 5. They recognize the efficacy of using this strategy when the quotient of a division equation is greater than 5, and also realize that the dividend must be decomposed into numbers that are divisible by the divisor. For example, to solve $64 \div 8$, 64 can be decomposed as 40 and 24 because both are divisible by 8.

Topic D: Multiplication and Division Using Units of

- Nines are placed late in the module so that students have enough experience with multiplication and division to recognize, analyze, and apply the rich patterns found in the manipulation of these facts.

- Focuses on the study of patterns as they relate to the fact 9 = 10 – 1. Students discover that the tens digit in the product of a nines fact is 1 less than the multiplier, and that the ones digit in the product is 10 minus the multiplier. For example, 9 × 3 = 27, 2 = 3 – 1, and 7 = 10 – 3. They also see that the digits of nines facts products produce a sum of 9, as in the example above (2 + 7 = 9).

a. **9 × 6 =** ______



9 sixes = 10 sixes – 1 six

=_____ – 6

= _____

Topic E: Analysis of Patterns and Problem Solving Including Units of 0 and 1

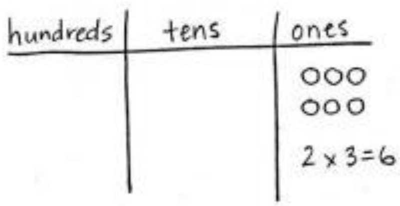
- Students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is among the more complex, particularly as it relates to division.

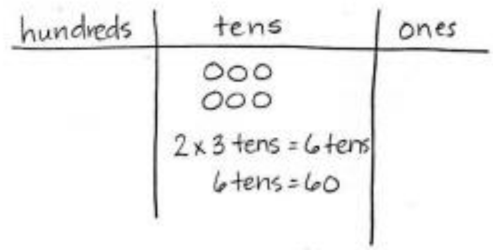
- They use patterns to understand that n × 0= 0, and show why the result of dividing a number by 0 is undefined, but that dividing 0 by another number results in 0

Topic F: Multiplication of Single-Digit Factors and Multiples of 10

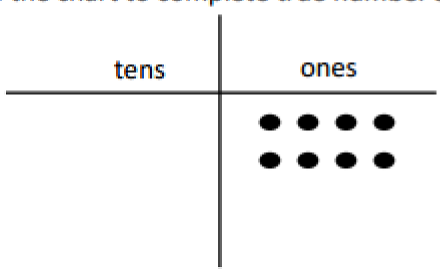
- Students initially use the place value chart to multiply by multiples of 10. To solve 2 × 40, for example, they begin by modeling 2 × 4 in the ones place. Students relate this to multiplying 2 × 4 tens, locating the same basic fact in the tens column. They see that when multiplied by 10, the product shifts one place value to the left. Complexities are addressed as regrouping becomes involved with problems like 4 × 6, where the product has mixed units of tens and ones. However, the same principle applies—the digits shift once to the left.

- To solve a fact like 2 × 30, they first model the basic fact 2 × 3 on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10: 2 × 3 tens can be found by simply locating the same basic fact in the tens column.





Use the chart to complete true number sentences.



a. 2 × 4 ones = _______ ones

2 × 4 = _______

- place value understanding becomes more abstract as students model place value strategies using the associative property

  2 × 30 = 2 × (3 × 10) = (2 × 3) × 10.